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Visibility for anharmonic fringes

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Abstract

We examine a new approach to the visibility of interference fringes defined as the distance L^2 to the uniform distribution. We show that this approach is superior to the standard approach when examining anharmonic fringes, while they coincide for harmonic fringes. We demonstrate the close relationship between this formalism and other well-studied measures of quantum uncertainty and information. We also show that this approach preserves the connection between visibility and correlation functions irrespective of the complexity of the interferometric pattern.

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1. Introduction

Interference is a basic phenomenon that occupies a relevant position in many areas of physics. As a matter of fact, after the introduction of quantum theory, interference is a phenomenon that can be displayed by all physical systems. This fundamental characteristic along with its wide range of application have led to increasingly sophisticated implementations of interference that are far from the classic examples [1, 2]. This framework has been further enlarged by the thorough revision of fundamental concepts caused by the emergence of quantum theory. This is the case of the phase difference variable whose proper quantum description and measurement is still the subject of vivid controversy and active research [3]. These facts have prompted an intense scrutiny of the information conveyed by general interference patterns and how can it be properly measured.

Despite the dramatic growth of the subject, interference is most often analysed using classic tools, such as the visibility, devised for the most simple observations of this phenomenon in the form of harmonic fringes. As we show here, these tools may fail when applied to anharmonic patterns. Moreover, in the most general case the fruitful connection between the standard definition of visibility and correlation functions is lost. This lack of a proper generalization may be a source of misunderstandings as revealed by recent works in quantum interference that suggest the redefinition of basic interferometric concepts [2, 4, 5].

In this paper, we focus on the idea of visibility as a measure of the amount of interference. Interference is always identified as the alteration of a uniform or featureless distribution. Therefore it seems natural to define the visibility as the distance to the uniform distribution. Among the different possibilities available we focus on the L^2 distance. This choice is justified since it leads to meaningful results, it is closely related to other approaches to similar problems and also because it leads to simple calculations. This distance has already been used in the context of wave–particle duality for harmonic fringes produced by pairs of beams of a multiple beam interferometer [4].

The main goal of this paper is to demonstrate that this new definition of visibility has better properties than the classic approach. This is shown by means of basic relevant examples of interference. We show that the new definition is always sensitive to the information content of the fringes while the classic definition is rather insensitive. We show that the new definition preserves the fruitful relation between visibility and correlation functions that the classic definition lacks for anharmonic fringes. Finally we show that the new approach can be recognized as a meaningful measure of fluctuations and information already used in other contexts. Nothing similar is possible for the classic definition.

2. Visibility

We represent a general interference pattern by a real function $I(\phi)$. The actual meaning of I and the way it is measured are fully arbitrary. From a quantum perspective we have

$$I(\phi) \propto \mathrm{tr}[\rho\Delta(\phi)] \tag{2.1}$$

where ρ is the density matrix representing the state of the system and $\Delta(\phi)$ is a family of operators that depend on the interferometric arrangement and on the measuring strategy. Note that it is not excluded that $\Delta(\phi)$ may depend even on ρ as in the experiment carried out in [1]. For definiteness, in this paper we focus on interference patterns fully parametrized by a single bounded phase variable ϕ defined in a 2π interval $\phi \in [-\pi, \pi)$. This is a very common situation and usually ϕ represents phase difference. Nevertheless, it will be clear that all formulae admit straightforward generalizations to an arbitrary number of variables with arbitrary ranges of variation. For definiteness and without loss of generality, we assume that the featureless uniform distribution that corresponds to total lack of interference is a constant not depending on ϕ that will be denoted as $\langle I\rangle$.

2.1. Definitions

The classic definition of visibility is [6]

$$V_{\mathrm{class}} = \frac{I_{\mathrm{max}} - I_{\mathrm{min}}}{I_{\mathrm{max}} + I_{\mathrm{min}}} \tag{2.2}$$

where I_{max} and I_{min} are the local or global maximum and minimum of $I(\phi)$, respectively.

On the other hand, we can also define the visibility $\mathcal{V}$ as the distance between $I(\phi)$ and the uniform distribution $\langle I\rangle$ [4]

$$\mathcal{V}^2 = \frac{1}{2\pi\langle I\rangle^2}\int_{2\pi} \mathrm{d}\phi [I(\phi) - \langle I\rangle]^2 = \frac{1}{\langle I\rangle^2}(\langle I^2\rangle - \langle I\rangle^2) \tag{2.3}$$

where

$$\langle I \rangle = \frac{1}{2\pi} \int_{2\pi} \mathrm{d}\phi I(\phi) \qquad \langle I^2 \rangle = \frac{1}{2\pi} \int_{2\pi} \mathrm{d}\phi [I(\phi)]^2. \tag{2.4}$$

As mentioned in the introduction we use the standard L^2 distance. In the next section, we show examples demonstrating that $\mathcal{V}$ is not bounded from above, $\infty > \mathcal{V} \geqslant 0$. On the other hand, $\mathcal{V} = 0$ if and only if $I(\phi) = \langle I \rangle$ for all ϕ. If a bounded expression for the visibility is preferred, we may define a normalized visibility V as

$$V^2 = \frac{\mathcal{V}^2}{1 + \mathcal{V}^2} = \frac{1}{\langle I^2 \rangle} (\langle I^2 \rangle - \langle I \rangle^2). \tag{2.5}$$

It can be appreciated that the only difference between equations (2.3) and (2.5) is the denominator, $\langle I \rangle^2$ or $\langle I^2 \rangle$, respectively. For simplicity, in the following we refer exclusively to $\mathcal{V}$.

All these definitions are invariant under scale transformations $I(\phi) \to \lambda I(\phi)$ where λ is a constant. We have considered global definitions (integrals extended to a full 2π interval), but local definitions can be considered simply by suitably restricting the interval of integration in equations (2.3) and (2.4), and replacing 2π by the length of the interval.

2.2. Fourier domain

We can express $I(\phi)$ in terms of its Fourier components

$$I(\phi) = \sum_{k=-\infty}^{\infty} \tilde{I}_k \, \mathrm{e}^{\mathrm{i}k\phi} = \tilde{I}_0 + 2 \sum_{k=1}^{\infty} |\tilde{I}_k| \cos(k\phi + \delta_k) \tag{2.6}$$

where

$$\tilde{I}_k = \tilde{I}^*_{-k} = |\tilde{I}_k| \, \mathrm{e}^{\mathrm{i}\delta_k} = \frac{1}{2\pi} \int_{2\pi} \mathrm{d}\phi \, \mathrm{e}^{-\mathrm{i}k\phi} I(\phi). \tag{2.7}$$

With the help of this decomposition we obtain a useful expression for the visibility

$$\mathcal{V}^2 = \sum_{k \neq 0} \mathcal{V}_k^2 = 2 \sum_{k=1}^{\infty} \mathcal{V}_k^2 \tag{2.8}$$

where

$$\mathcal{V}_k = \frac{|\tilde{I}_k|}{\tilde{I}_0} \tag{2.9}$$

and $\mathcal{V}_{-k} = \mathcal{V}_k$.

From a quantum perspective we can express $\mathcal{V}_k$ in equations (2.8) and (2.9) as

$$\mathcal{V}_k = \frac{|\langle \psi | E_k | \psi \rangle|}{\langle \psi | E_0 | \psi \rangle} \tag{2.10}$$

where $|\psi\rangle$ represents the state of the system, assumed pure for simplicity, and E_k are the operators

$$E_k = \int_{2\pi} \mathrm{d}\phi \, \mathrm{e}^{\mathrm{i}k\phi} \Delta(\phi). \tag{2.11}$$

It is worth pointing out that no relations equivalent to equations (2.8) and (2.9) can be derived from the classic definition (2.2). The relevance and usefulness of these relations are demonstrated below.

2.3. Harmonic and anharmonic fringes

Let us show that the above expressions in the Fourier domain imply that formulae (2.2) and (2.3) coincide for harmonic fringes

$$I(\phi) = \tilde{I}_0 + 2|\tilde{I}_k| \cos(k\phi + \delta_k). \tag{2.12}$$

In this case $\mathcal{V}_{k'} = 0$ for $k' \neq \pm k, 0$ and we obtain

$$V_{\mathrm{class}} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = 2\frac{|\tilde{I}_k|}{\tilde{I}_0} = 2\mathcal{V}_k = \sqrt{2}\mathcal{V}. \tag{2.13}$$

Thus, equations (2.2) and (2.3) give essentially the same result for harmonic fringes.

Concerning anharmonic fringes we can appreciate that equations (2.2) and (2.3) lead in general to different results. This can easily be proven by considering a distribution of the form $I(\phi) = \tilde{I}_0 + 2|\tilde{I}_1| \cos(\phi) + 2|\tilde{I}_2| \cos(2\phi + \delta_2)$. It can be seen that V_{class} depends on δ_2 while $\mathcal{V}$ does not. Further relevant examples of the discrepancy between $\mathcal{V}$ and V_{class} are examined in section 3.

2.4. Correlation functions

To the best of our knowledge, for anharmonic fringes there is no relation between the classic definition (2.2) and correlation functions. (We should stress that, by definition, we are excluding the case of local approximations of anharmonic fringes by harmonic ones.) However we can show that such a relation is possible using the definition examined in this paper. The key point is equation (2.8) that expresses $\mathcal{V}$ as a function of the visibility for the harmonic Fourier components of $I(\phi)$. The concrete statistical meaning of $\mathcal{V}$ depends on the experimental arrangement and also on the nature of the interfering system. We illustrate this point with some examples.

We can start by considering the interference of two electromagnetic field modes of the same frequency ω. For the standard case of balanced intensity measurements at the output of two-beam interferometers (such as Young, Michelson and Mach–Zehnder) we have

$$\Delta(\phi) = \mathrm{e}^{-\mathrm{i}\phi} a_1^\dagger a_2 + \mathrm{e}^{\mathrm{i}\phi} a_1 a_2^\dagger \tag{2.14}$$

where a_1 and a_2 are the complex amplitude operators for the internal modes of the interferometer, and ϕ represents the phase difference between a_1 and a_2. In this case of harmonic fringes we have that $\mathcal{V}_k = 0$ for $k \neq \pm 1, 0$ and then

$$E_1 \propto a_1^\dagger a_2 \qquad \mathcal{V}_1 \propto \left|\langle a_1^\dagger a_2\rangle\right|. \tag{2.15}$$

These equations can be easily generalized to non-monochromatic multimode fields leading to non-harmonic fringes. In such a case, it can be seen that $\mathcal{V}_k$ is proportional to the mutual spectral density of the interfering fields [6]. Therefore, $\mathcal{V}^2$ is the integral of the squared modulus of the mutual spectral density.

Due to the generality of the approach developed here we can present more involved examples far from the familiar interferometric arrangements examined above. For example we can focus on the usual situation in which $\Delta(\phi)$ can be expressed as

$$\Delta(\phi) = |\phi\rangle\langle\phi| \tag{2.16}$$

for some vectors $|\phi\rangle$. In this case we have that

$$E_k = \sum_n |n\rangle\langle n + k| \qquad \langle\psi|E_k|\psi\rangle = \sum_n \psi^*(n)\psi(n + k) \tag{2.17}$$

where $|n\rangle$ is the basis dual to $|\phi\rangle$

$$|\phi\rangle = \frac{1}{\sqrt{2\pi}} \sum_n e^{in\phi}|n\rangle \qquad |n\rangle = \frac{1}{\sqrt{2\pi}} \int_{2\pi} d\phi \, e^{-in\phi}|\phi\rangle \tag{2.18}$$

and $\psi(n) = \langle n|\psi\rangle$. We can see that, in this case, E_k is the autocorrelation function of the wavefunction $\psi(n)$ so that $\mathcal{V}^2$ is the integral of the its squared modulus.

These examples show that for each situation we can find a direct relationship between $\mathcal{V}$ and suitable correlation functions and spectral densities. We stress that no relations of this form can be derived from equation (2.2).

2.5. Uncertainty measures

There are many situations in which I is actually a probability distribution $I(\phi) = P(\phi)$ so that

$$\mathcal{V}^2 = 2\pi \int d\phi [P(\phi)]^2 - 1. \tag{2.19}$$

This and similar relations have already been used as measures of localization and uncertainty [7–11]. In this context $1/\mathcal{V}$ can be interpreted as the effective area where $P(\phi)$ is different from the uniform constant value $1/(2\pi)$. More specifically, equation (2.3) can be regarded as a particular case of a general class of measures of localization [10–12]

$$M_r = \left(\int d\phi [P(\phi)]^{1+r} \right)^{1/r}. \tag{2.20}$$

In our case we have that $\mathcal{V}^2 \propto M_1$. This identification endows the definition (2.3) with desirable properties such as those listed in [10].

In this same context, the mean values $|\langle\psi|E_k|\psi\rangle|$ can be regarded as a measure of fluctuations of the observable $\Delta(\phi)$ (which we may call certainty [13]) closely related to the dispersion [14–16]. In this sense the visibility (2.3) is a sum of certainties as revealed by equations (2.8) and (2.10).

This natural relation between visibility and fluctuations has found applications in the context of quantum complementarity exemplified by the wave–particle duality. In this context the visibility is intended to provide a measure of the degree of wave behaviour (represented by the observable $\Delta(\phi)$) of the interfering system. We have just shown that $\mathcal{V}$ is perfectly suited for this purpose. Moreover, in a previous paper [13] this identification is used to introduce suitable duality relations involving $\mathcal{V}$ that explain how complementarity is enforced by the quantum fluctuations of the measuring apparatus for two-dimensional systems. In [13] these duality relations are examined and compared to similar relations involving V_{class} [17]. The duality relations based on $\mathcal{V}$ can be generalized to systems of arbitrary dimension, as shown in [9]. The usefulness of $\mathcal{V}$ in this context is further demonstrated in [4].

3. Applications

In this section we apply the above ideas to some relevant fringe patterns comparing the definitions (2.2) and (2.3). The results of the comparison are further discussed in section 4. Focusing on anharmonic fringes we examine interferometric patterns made of isolated peaks in uniform backgrounds. This is a very representative case directly related to many experimental arrangements including classic multiple-beam arrangements, such as the Fabry–Perot interferometer, as well as recent and sophisticated examples of quantum measurement, such as that reported in [1].

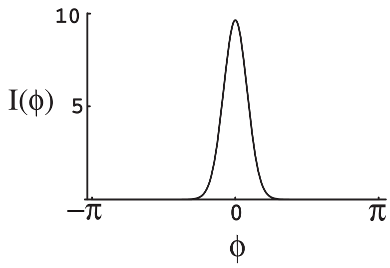

Figure 1. Bright peak in a dark background for $\lambda = 15$ and $\alpha = 1$ in equation (3.1).

3.1. Harmonic fringes

The first example we may consider is the standard case of the harmonic patterns in equation (2.12) produced by two-beam interferometers. We have already examined this case in section 2.3 showing that both definitions of visibility provide the same result. In particular, equation (2.13) implies that for harmonic fringes $\mathcal{V}$ is bounded from above $\mathcal{V} \leqslant 1/\sqrt{2}$.

3.2. Anharmonic fringes

For the sake of definiteness we focus on patterns $I(\phi)$ having a single narrow peak centred at $\phi = 0$ in a uniform background. As a model for such a function for a periodic variable we use the von Mises distribution proportional to $\exp(\lambda \cos\phi)$ [18]. Using the invariance under scale changes we normalize the distributions on the form $\int_{2\pi} \mathrm{d}\phi I(\phi) = 2\pi$ such that $\langle I \rangle = 1$. In these conditions

$$I(\phi) = 1 - \alpha \left(1 - \frac{1}{\mathcal{I}_0(\lambda)} \mathrm{e}^{\lambda \cos\phi} \right) \tag{3.1}$$

where $\mathcal{I}_0(\lambda)$ is the modified Bessel function of order zero, and α and λ are real parameters that determine the height of the background and the width of the peak, respectively. Incidentally, the von Mises function is the probability distribution for the quantum phase of a single-mode field prepared in a type of phase-number intelligent state [15, 19].

The function (3.1) is the analogue of a Gaussian for periodic variables (other analogues can be found in [16, 20]). This is especially clear in the limit $\lambda \gg 1$. In such a case $\exp(\lambda \cos\phi)$ can be safely replaced by a Gaussian dependence $\exp(-\lambda\phi^2/2)$ extending the range of variation of ϕ to $\pm\infty$.

For the distribution (3.1) we have

$$\mathcal{V}^2 = \alpha^2 \left(\frac{\mathcal{I}_0(2\lambda)}{\mathcal{I}_0^2(\lambda)} - 1 \right). \tag{3.2}$$

Since we always consider narrow enough peaks we assume $\lambda \gg 1$. In such a case $\mathcal{I}_0(x) \simeq \exp(x)/\sqrt{2\pi x}$ for $x \gg 1$ so that $\mathcal{V}$ can be approximated as

$$\mathcal{V}^2 \simeq \alpha^2 \sqrt{\pi\lambda}. \tag{3.3}$$

Incidentally this demonstrates that for anharmonic fringes $\mathcal{V}$ is not bounded from above.

3.2.1. Bright peak in a dark background. Let us consider the case of a bright peak in a dark background, as illustrated in figure 1. This corresponds to $\alpha = 1$ in equation (3.1) so that for large λ we have from equation (3.3) $\mathcal{V}^2 \simeq \sqrt{\pi\lambda}$ and $\mathcal{V} \to \infty$ when $\lambda \to \infty$. In other words, $\mathcal{V}$ increases without limit when the width of the peak decreases. This agrees well with

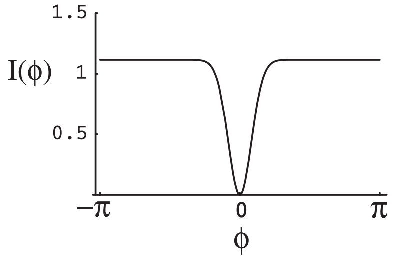

Figure 2. Dark peak in a bright background for $\lambda = 15$.

the fact that this type of interference provides better resolution (i.e. resolving power) than the harmonic fringes. Moreover, $\mathcal{V}$ clearly reflects that the resolution depends on the width of the peak (i.e. the finesse).

On the other hand, for $\lambda \gg 1$ we always have $V_{\text{class}} \simeq 1$ irrespective of the width of the peak. More precisely $V_{\text{class}} = \tanh \lambda$. Therefore $\mathcal{V}$ is far more sensitive than V_{class} to the phase information conveyed by the interference pattern.

It is worth noting that for $\alpha = 1$ and $\lambda \gg 1$ the distribution (3.1) corresponds to the quantum phase of states with a Gaussian distribution of photon number [15]. In such a case $\lambda = (\Delta n)^2$, where Δn is the uncertainty in photon number. This is the case of intense coherent states and quadrature squeezed states of large mean photon number and moderate squeezing [21].

3.2.2. Dark peak in a bright background. Let us consider the case of a dark peak $I(\phi = 0) = 0$ in a bright uniform background, as illustrated in figure 2. This corresponds to $\alpha = \mathcal{I}_0(\lambda)/[\mathcal{I}_0(\lambda) - \exp(\lambda)]$. In this case for $\lambda \gg 1$ we have from equation (3.3) $\mathcal{V}^2 \simeq 1/(2\sqrt{\pi\lambda})$ so that $\mathcal{V} \to 0$ when $\lambda \to \infty$. Thus the visibility tends to zero when the peak narrows. On the other hand, the classic definition gives the opposite result; always maximum visibility $V_{\text{class}} = 1$ irrespective of the width of the peak.

Let us show that also in this case $\mathcal{V}$ provides the right answer. As the width of the peak decreases, the area it encloses also decreases. This implies that $I(\phi)$ approaches the uniform distribution. In other words, as $\lambda \to \infty$ it is more and more difficult to detect the existence of the peak. This is especially clear if we focus on a quantum realization where the area enclosed by the peak represents the probability of the occurrence of a certain effect. If the area decreases, the occurrence of a meaningful outcome tends to have a null probability so that it will be accordingly difficult to detect the existence of any interference effect.

3.2.3. Bright peak in a bright background. A bright peak in a less bright background corresponds to $1 > \alpha > 0$ in equation (3.1) so that $1 - \alpha$ represents the height of the background (figure 3). In order to investigate the differences between V_{class} and $\mathcal{V}$ let us consider the case in which $\alpha^2\sqrt{\lambda} \to 0$ and $\alpha\sqrt{\lambda} \to \infty$ when $\lambda \to \infty$. This corresponds to a peak whose height tends to infinity but whose width and the area it encloses tend to zero. In such a case it can be easily seen that V_{class} and $\mathcal{V}$ lead again to opposite results: $V_{\text{class}} \to 1$ while $\mathcal{V} \to 0$.

We can show that also in this case $\mathcal{V}$ provides a more meaningful measure of the visibility of the interference. The limit $V_{\text{class}} \to 1$ occurs because $I_{\max} \to \infty$ while $I_{\min} \to 1$. However, the area enclosed by the peak tends to zero so that the interference pattern tends to be uniform. Therefore, the same reasoning of the previous example applies here.

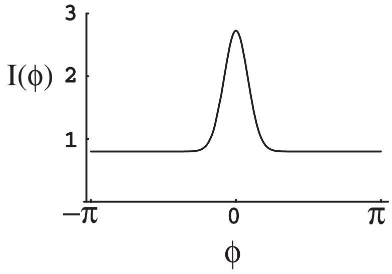

Figure 3. Bright peak in a bright background for $\lambda = 15$ and $\alpha = 0.2$ in equation (3.1).

3.2.4. Reciprocal peak likelihood phase states. The results of the preceding examples suggest that $\mathcal{V}$ may provide a suitable measure of phase resolution better behaved than V_{class}. In this subsection we deepen this issue focusing on a specific approach to the quantum measurement of the phase shift of a single mode field. Our aim is to apply and compare both definitions of visibility, discussing the relevance of the respective conclusions.

A variety of different approaches conclude that there is a fundamental quantum limit for the resolution $\delta\phi$ of a phase-shift measurement depending on the fluctuations of the photon number Δn on the form $\delta\phi \propto 1/\Delta n$. The ultimate resolution achievable (the Heisenberg limit) occurs when $\Delta n = \bar{n}$, where $\bar{n}$ is the mean number of photons involved in the measurement [3, 15, 22]. It is worth noting that the Heisenberg limit can be reached by the states with Gaussian photon number distribution mentioned in subsection 3.2.1 provided that $\lambda = (\Delta n)^2 = \bar{n}^2 \gg 1$.

The actual performance of a measurement depends on the measure of error adopted. In [23] a phase measurement is proposed that seemingly would break the Heisenberg limit with a resolution scaling as $\delta\phi \propto 1/\bar{n}^2$ or even better. The statistics of such phase measurement is given by projection of an optimum input state $|\psi\rangle$ on the Susskind–Glogower phase states $|\phi\rangle$ (ideal phase measurement), $P(\phi) = |\langle\phi|\psi\rangle|^2$, where

$$|\phi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} \mathrm{e}^{\mathrm{i}n\phi} |n\rangle \tag{3.4}$$

and

$$|\psi\rangle = \mathcal{N} \sum_{n=0}^{M} \frac{1}{r+n} |n\rangle. \tag{3.5}$$

Here M and r are state parameters and $\mathcal{N}$ is a normalization constant. The uncertainty criterion adopted in [23] is the reciprocal peak likelihood $\delta\phi = 1/P_{\max}$ where $P_{\max}$ is the maximum of $P(\phi)$. However it has been shown that reciprocal peak likelihood is not a meaningful performance measure in this context [24]. Other data analyses imply that $\delta\phi$ actually does not scale as $1/\bar{n}^2$ so that this strategy does not reach the Heisenberg limit [24].

Our goal here is to apply both definitions of visibility to the distribution $P(\phi) = |\langle\phi|\psi\rangle|^2$ studying which definition gives more meaningful results in the light of the above discussion. In general, for fixed r we find that both visibilities grow when the mean photon number increases. However they grow at a different rate, as can be seen in figure 4. In these plots we have compared $\mathcal{V}$ and V_{class} for the state (3.5) with the corresponding visibilities $\mathcal{V}'$ and V'_{class} for a photon-number Gaussian state with the same mean number of photons as (3.5) and with $\Delta n = \bar{n}$ (i.e. the states in subsection 3.2.1 that reach the Heisenberg limit).

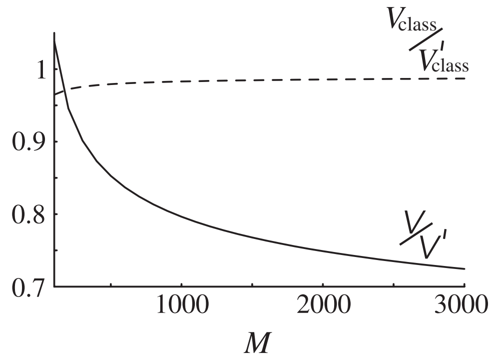

Figure 4. Plot of $\mathcal{V}/\mathcal{V}'$ (solid curve) and V_{class}/V'_{class} (dashed curve) as a function of M, where $\mathcal{V}$ and V_{class} are the visibilities for the phase probability distribution of the state (3.5) with $r = 1$ while $\mathcal{V}'$ and V'_{class} are the visibilities for the phase probability distribution for a Gaussian state reaching the Heisenberg limit (i.e. $\alpha = 1, \lambda = \bar{n}^2$ in equation (3.1)).

In figure 4 we have represented $\mathcal{V}/\mathcal{V}'$ (solid curve) and V_{class}/V'_{class} (dashed curve) as functions of M for $r = 1$, and it must be taken into account that $\bar{n}$ increases when M increases. We can appreciate that for the classical definition $V_{class} \simeq V'_{class}$. This implies that for increasing M the states (3.5) would provide the same visibility as the states reaching the Heisenberg limit.

On the other hand, it can be seen in figure 4 that the use of $\mathcal{V}$ leads to opposite results and for the state (3.5) the visibility $\mathcal{V}$ is increasingly worse than $\mathcal{V}'$ when M increases. This result coincides with the conclusions of [24] and confirms that $\mathcal{V}$ is more sensitive than V_{class} to the relevant details of the interference.

4. Discussion

In this paper, we have examined a suitable definition of visibility. The analysis in section 2 and the examples analysed above demonstrate that this new approach is better behaved than the classic approach. Some of the difficulties that the classical definition encounters arise because it puts too much emphasis on the closeness of I_{min} to zero. For example, when $I_{min} = 0$ we have $V_{class} = 1$ irrespective of any other characteristic of the fringes. We have shown that this implies that V_{class} is, to a large extent, insensitive to the information conveyed by the interference pattern.

More specifically, we have presented some examples of interference patterns that are arbitrarily close to the uniform distribution for which the classic definition gives the inconsistent result $V_{class} = 1$ while the new definition provides a meaningful value $\mathcal{V} = 0$. The other examples confirm that $\mathcal{V}$ is better behaved since it properly accounts for the following facts: (a) narrow peaks can provide a much more efficient interference effect than harmonic fringes (as corroborated by the resolving power of multiple-beam interferometers); (b) the quality and usefulness of the interference depends not only on the height of peaks and valleys but also (and more importantly) on the width of the peak and on the area it encloses. We have shown that V_{class} is insensitive to these features while $\mathcal{V}$ clearly reflects all of them. These are items of practical importance since the primary goal of most interferometric arrangements is to provide information about the interfering system. In this same context, we have shown that $\mathcal{V}$ is closely related to other well-studied measures of fluctuations and information. This can explain its sensitivity to the amount of information provided by the interference.

It is known that for the harmonic fringes obtained by intensity measurements the classic visibility is proportional to correlation functions. However, for more involved situations such a powerful connection is lost and no general relation of this kind is known. The fruitful connection between visibility and correlation functions is successfully recovered by the definition studied in this paper.

All these points demonstrate that $\mathcal{V}$ is a natural extension of the concept of visibility including arbitrary anharmonic interference patterns.

References

[1] Mei M and Weitz M 2001 *Phys. Rev. Lett.* **86** 559
Luis A 2001 *J. Phys. A: Math. Gen.* **34** 8597
[2] Björk G, Inoue S and Söderholm J 2001 *Phys. Rev.* A **62** 023817
[3] Luis A and Sánchez-Soto L L 2000 *Progress in Optics* vol 41 (Amsterdam: Elsevier) p 421
[4] Dürr S 2001 *Phys. Rev.* A **64** 042113
[5] Gennaro G, Leonardi C, Lillo F, Vaglica A and Vetri G 1994 *Opt. Commun.* **112** 67
[6] Born M and Wolf E 1980 *Principles of Optics* (Oxford: Pergamon)
Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)
[7] Heller E J 1987 *Phys. Rev.* A **35** 1360
Bialynicki-Birula I, Freyberger M and Schelich W 1993 *Phys. Scr.* T **48** 113
Lukš A and Peřinová V 1994 *Quantum Opt.* **6** 125
Sugita A and Aiba H 2001 *Preprint* nlin.CD/0106012
Mirbach B and Korsch H J 1998 *Ann. Phys., NY* **265** 80
Hall M J W 1999 *Phys. Rev.* A **59** 2602
[8] Brukner Č and Zeilinger A 1999 *Phys. Rev. Lett.* **83** 3354
Řeháček J and Hradil Z 2002 *Phys. Rev. Lett.* **88** 130401
[9] Larsen U 1990 *J. Phys. A: Math. Gen.* **23** 1041
[10] Maassen H and Uffink J B M 1988 *Phys. Rev. Lett.* **60** 1103
[11] Gnutzmann S and Życzkowski K 2001 *J. Phys. A: Math. Gen.* **34** 10123
[12] Wehrl A 1978 *Rev. Mod. Phys.* **50** 221
Wehrl A 1991 *Rep. Math. Phys.* **30** 119
[13] Luis A 2001 *Phys. Rev.* A **64** 012103
[14] Lévy-Leblond J M 1976 *Ann. Phys., NY* **101** 319
Breitenberger E 1985 *Found. Phys.* **15** 353
Uffink J B M 1985 *Phys. Lett.* A **108** 59
Lévy-Leblond J-M 1985 *Phys. Lett.* A **111** 353
Hradil Z 1992 *Phys. Rev.* A **46** R2217
Hradil Z 1992 *Quantum Opt.* **4** 93
[15] Peřinová V, Lukš A and Peřina J 1998 *Phase in Optics* (Singapore: World Scientific)
[16] Opatrný T 1994 *J. Phys. A: Math. Gen.* **27** 7201
[17] Greenberger D M and Yasin A 1988 *Phys. Lett.* A **128** 391
Jaeger G, Shimony A and Vaidman L 1995 *Phys. Rev.* A **51** 54
Englert B-G 1996 *Phys. Rev. Lett.* **77** 2154
Dürr S, Nonn T and Rempe G 1998 *Nature* **395** 33
Dürr S, Nonn T and Rempe G 1998 *Phys. Rev. Lett.* **81** 5705
Schwindt P D D, Kwiat P G and Englert B-G 1999 *Phys. Rev.* A **60** 4285
Abranyos Y, Jakob M and Bergou J 2000 *Phys. Rev.* A **61** 013804
Dürr S and Rempe G 2000 *Am. J. Phys.* **68** 1021
Trifonov A, Björk G, Söderholm J and Tsegaye T 2002 *Eur. Phys. J.* D **18** 251
[18] Rao R C 1973 *Linear Statistical Inference and its Applications* (New York: Wiley)
[19] Lukš A and Peřinová V 1992 *Phys. Rev.* A **45** 6710
[20] Klimov A B and Chumakov S M 1997 *Phys. Lett.* A **235** 7
[21] Peřina J 1991 *Quantum Statistics of Linear and Nonlinear Optical Phenomena* (Dordrecht: Kluwer)
[22] Ou Z Y 1997 *Phys. Rev.* A **55** 2598

[23] Shapiro J H, Shepard S R and Wong N C 1989 *Phys. Rev. Lett.* **62** 2377
Shapiro J H and Shepard S R 1991 *Phys. Rev.* A **43** 3795
[24] Braunstein S L, Lane A S and Caves C M 1992 *Phys. Rev. Lett.* **69** 2153
Braunstein S L 1992 *Phys. Rev. Lett.* **69** 3598
Lane A S, Braunstein S L and Caves C M 1993 *Phys. Rev.* A **47** 1667
Hradil Z and Shapiro J 1992 *Quantum Opt.* **4** 31